KIT Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
High Performance Humanoid Technologies (H^2T)

Robotics I, WS 2024/2025

Exercise Sheet 5

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Date: January 22, 2025

Exercise 1 (Friction triangles)

Let $\mathbf{c} = (4, 3)^\top$ be the center of mass of a two-dimensional object shown in Figure 1. In the following, point contacts with friction are assumed. The contact forces are represented by friction triangles.

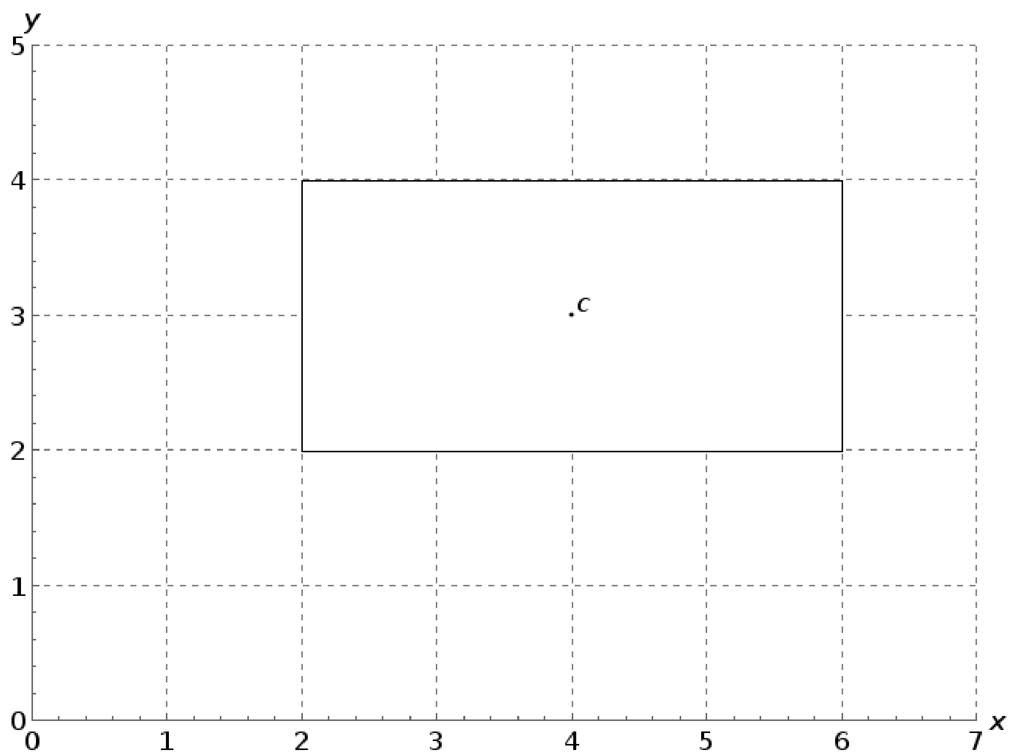

Figure 1: A two-dimensional object with center of mass $\mathbf{c}$.

1. Calculate the opening angle α of a friction triangle for the friction coefficient $\mu = 1$.
2. Let $\mathbf{p_1} = (3, 4)^\top$, $\mathbf{p_2} = (5, 2)^\top$ and $\mathbf{p_3} = (3, 2)^\top$ be contact points and let f_1, f_2 and f_3 be the corresponding force vectors.

$$\mathbf{f_1} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, \quad \mathbf{f_2} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}, \quad \mathbf{f_3} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$$

Draw the force vectors and the corresponding friction triangles at the contact points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$.

3. Determine the two force vectors at the edges of the friction triangles.

Exercise 2 (Grasp Wrench Space)

Let $\mathbf{c} = (4,3)^\top$ be the center of mass of a two-dimensional object shown in Figure 2 and let $\mathbf{p_1} = (3,4)^\top$, $\mathbf{p_2} = (5,2)^\top$ and $\mathbf{p_3} = (3,2)^\top$ be contact points. The contact forces are as follows: $\mathbf{f}_{a,1} = (0.5,-0.5)^\top, \mathbf{f}_{b,1} = (-0.5,-0.5)^\top, \mathbf{f}_{a,2} = \mathbf{f}_{a,3} = (-0.5,0.5)^\top, \mathbf{f}_{b,2} = \mathbf{f}_{b,3} = (0.5,0.5)^\top$.

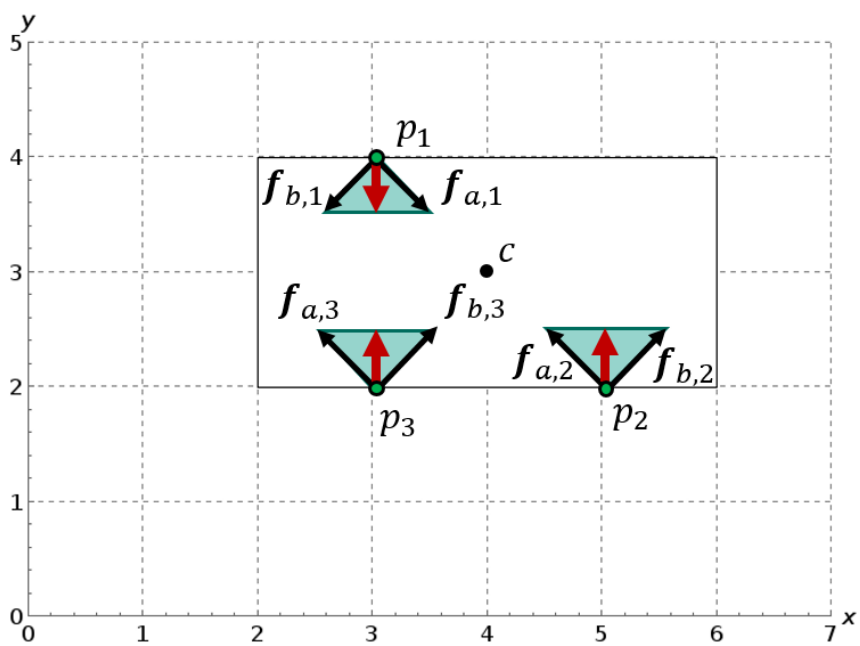

Figure 2: A two-dimensional object with center of mass $\mathbf{c}$.

1. Calculate the *wrenches* resulting from the contacts in 2D.

   Hint: In the two-dimensional case, the moment τ generated by a contact force $\mathbf{f}$ is a scalar that is calculated as follows: $\tau = \mathbf{d} \times \mathbf{f} = d_x \cdot f_y - d_y \cdot f_x$, where $\mathbf{d}$ is the vector from the center of mass to the contact point.

2. In Figure 3, draw the projection of the *Grasp Wrench Space* onto the (f_y, τ) plane for the contact points $\mathbf{p_1}$ and $\mathbf{p_2}$.

3. In Figure 4, draw the projection of the *Grasp Wrench Space* onto the (f_y, τ) plane for the contact points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$.

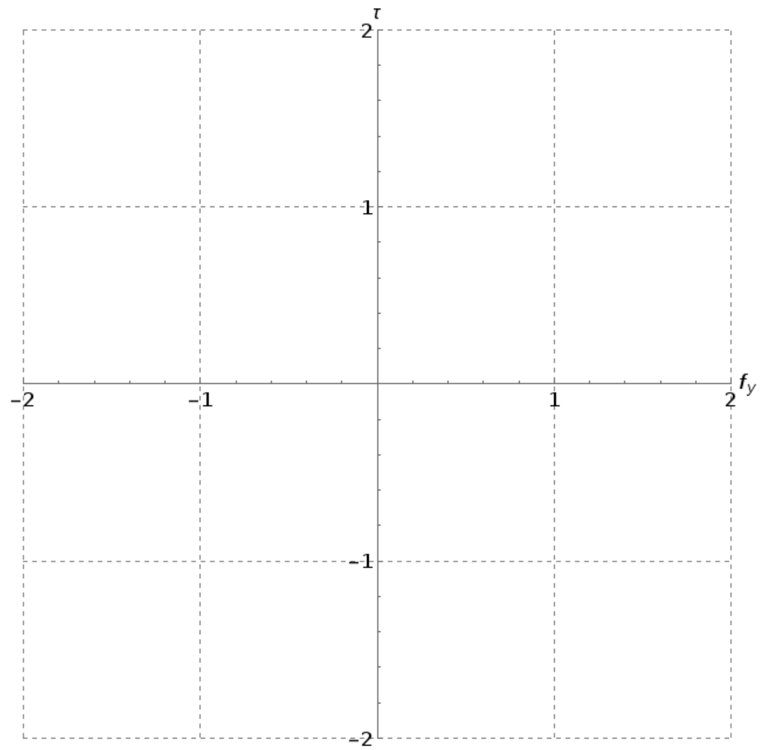

Figure 3: The dimensions f_y and τ of the Grasp Wrench Space.

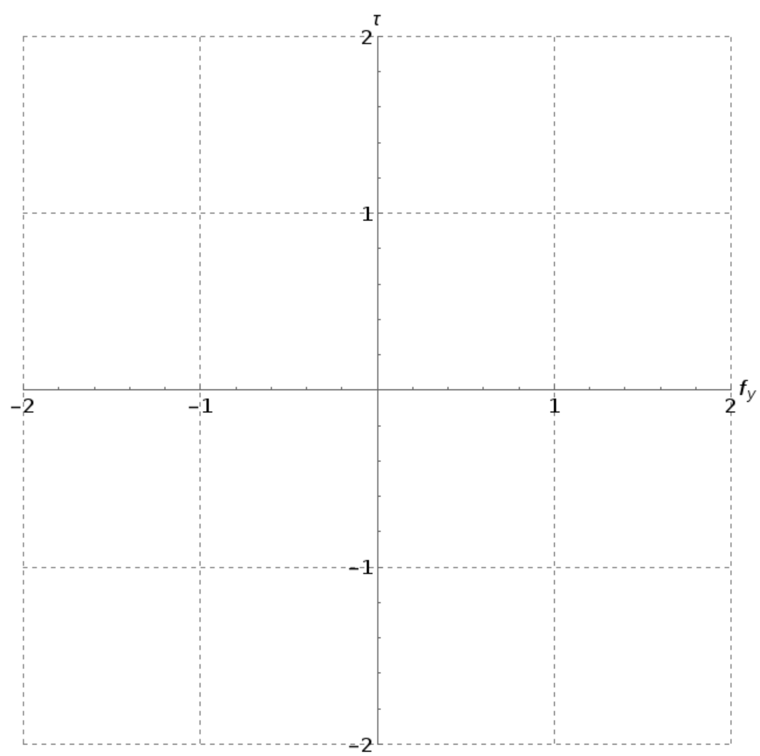

Figure 4: The dimensions f_y and τ of the Grasp Wrench Space.

Exercise 3 (Force closure)

Let $\mathbf{c} = (4, 3)^\top$ be the center of mass of a two-dimensional object shown in Figure 5. At the three contact points $\mathbf{p_1} = (3, 4)^\top$, $\mathbf{p_2} = (5, 2)^\top$ and $\mathbf{p_3} = (3, 2)^\top$, the following wrenches occur at the edges of the friction triangles:

$$\mathbf{w}_{a,1} = (0.5, -0.5, 0)^\top, \quad \mathbf{w}_{b,1} = (-0.5, -0.5, 1)^\top,$$
$$\mathbf{w}_{a,2} = (-0.5, 0.5, 0)^\top, \quad \mathbf{w}_{b,2} = (0.5, 0.5, 1)^\top,$$
$$\mathbf{w}_{a,3} = (-0.5, 0.5, -1)^\top, \quad \mathbf{w}_{b,3} = (0.5, 0.5, 0)^\top.$$

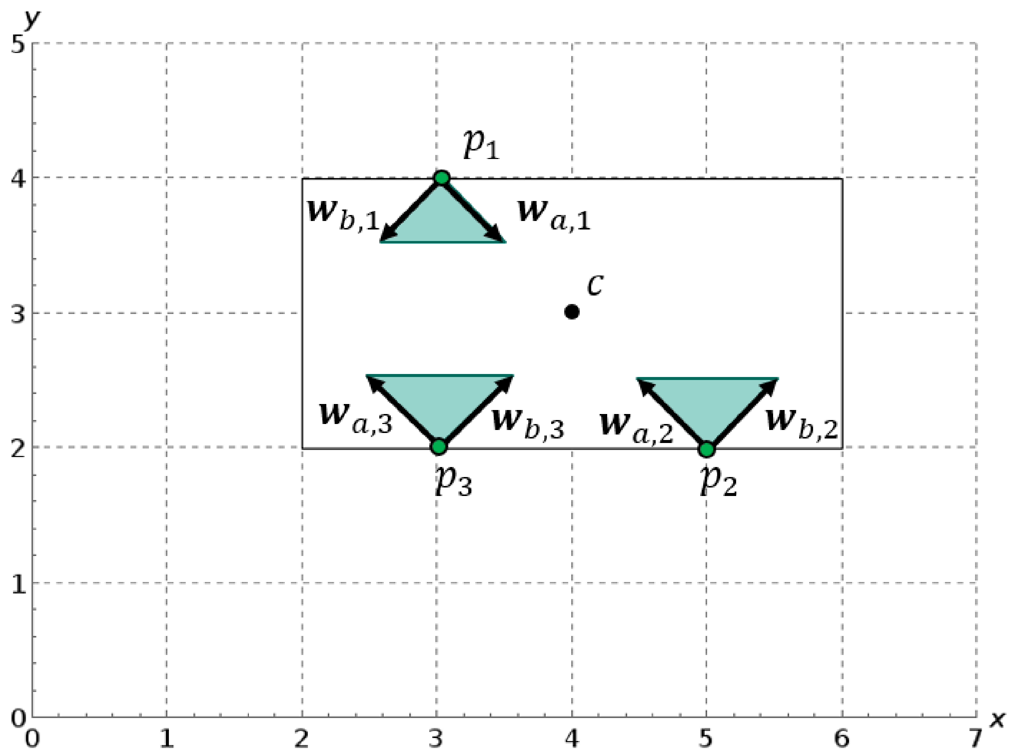

Figure 5: A two-dimensional object with center of mass $\mathbf{c}$.

1. Is the three-finger grasp at the points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$ force-closed? Justify your answer.
2. Is the two-finger grasp at the points $\mathbf{p_1}$ and $\mathbf{p_2}$ force-closed? Justify your answer.
3. How would you calculate the ε-metric for the two grasps? Specify whether ε is greater than, less than or equal to 0 for the two grasps.

Exercise 4 (Medial Axes)

The medial axis of a two-dimensional region $G \subset \mathbb{R}^2$ is the set of centers of the maximum circles in G. A circle K is a maximum circle in G if $K \subseteq G$ and if there is no circle K' for which $K \subset K' \subseteq G$ is true. Draw the medial axes of the regions $G_1, \ldots, G_5$.